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Article

Parameter with the Greatest Impact on the Mathematical Model of the Addiction of Drug Substances among Students of Tertiary Institutions in Nigeria

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Abstract

Drug addiction is a condition characterized by a self-destructive pattern of using a substance that leads to significant problems and distress which may include tolerance for or withdrawal from the drug substances. These drug abuse and addiction are both grouped as substance or drug use disorder. In this paper, a parameter with the greatest impact on the mathematical model of the addiction of drug substances among students of Nigerian tertiary institutions was determined. The obtained parameter with the greatest impact on the model was determined through the values of the eigenvalue elasticity and sensitivity analyses obtained via the computer program from the MATLAB software package. Out of the twelve parameters considered and used in the mathematical model, one parameter has the greatest impact based on the values obtained from the computer program. The parameter with the greatest impact on the mathematical model was determined to be the transmission rate of susceptible students into the class of illicit drug users which is represented by symbol β. This parameter was determined to assist the administrators of the tertiary institutions on how to effectively control, minimize and probably eradicate the addiction of drug substances among the students. Recommendations are given to reduce and minimize all forms of drug addiction among the students of tertiary institutions.

Keywords: Mathematical Model, Drug Substances, beta (β), MATLAB, Model Parameters, Eigenvalue Elasticity and Eigenvalue Sensitivity.

1. Introduction

Substance use disorder is a health condition involving compulsive substance use. It develops when substance use interferes with the ability to function day to day. It can occur with prescription or non-prescription drugs. Medical professionals previously used the term “drug abuse” to describe substance use disorder. Another term for substance use disorder is “addiction”. This differs from dependence on drugs prescribed to improve health issues [2].

The effects of drug use disorders on society are substantial. The cultivation of marijuana and production of synthetic drugs like methamphetamine has negative impact on soil and water supplies [4].

Individuals may abuse almost any substance whose ingestion can result in a euphoric ("high") feeling. While many are aware of the abuse of legal substances like alcohol or illegal drugs like marijuana (in most states) and cocaine, less well-known is the fact that inhalants like household cleaners and over-the-counter medications like cold medicines are some of the most commonly abused substances [7]. The following are many of the drugs and types of drugs that people commonly abuse and/or result in dependence: alcohol, amphetamines, anabolic steroids, caffeine, cannabis, cathinone (Balt salts) and cocaine. Other abuse drugs include ecstasy, hallucinogens, inhalants, nicotine, opiates, phencyclidine, sedative, hypnotic or antianxiety drugs [7].

According to Akanni et al (2021), drug is either synthetic or natural substance which causes biological, physiological, chemical and behavioural changes in the body when consumed by the users. Therefore, drug is a substance taken in either by injecting, smoking, inhaling, snorting, or swallowing which brings about modification to perceptions, cognition, mood, behaviour and general body functions [11].

In this paper, it is intended to determine the parameter that has the greatest impact on the mathematical model of drug addiction which incorporates drug user's class and addicted class on the mathematical modelling of the addiction of drug substances among students of tertiary institutions in Nigeria. Therefore, the study of the transmission dynamics of a SEDAR model is presented. The parameter is determined using the eigenvalue elasticity and sensitivity analyses [5] and this parameter will assist the government or tertiary institutions management to reduce the rate of addiction of drug substances among the students in the Nigerian tertiary institutions.

1.2 The Mathematical Model Formulation

The mathematical model is divided into a system of ordinary differential equations with five different sets in compartmental form depending on their substances uses status. The five compartments include the Susceptibles S(t) which comprise of all students that are at risk of using any substance (drug) at time t. All students who use any substance or drug of any form are grouped under drug users D(t). All students who are exposed to the use of the substance (drug) of any form are classified into the E(t) group. The compartment of all students who are addicted to the use of drugs are grouped under A(t) and those students who stopped using drugs either by abstinence or through rehabilitation or through parental guidance are under the class R(t). The rate at which the students are recruited into the susceptible class is Λ [3], the rate at which the students imitate their colleagues who use any substance is α and β is the transmission rate of the susceptible students are into the illicit drug users in the society while δ is the natural recovery rate of drug users and ϱ is the rate at which students return to the use of drugs. The natural death rate of all the students is denoted by μ and σ is the rate at which all drug users are addicted to the use of substances (drugs). The parameters τ and θ are the death rates of drug users and drug addicted respectively. The mathematical model is described by considering the dynamical equations for the student population. The mathematical model assumes that, those students who are recovered were still involved in the use of drug but not involved in the addiction of the drugs. The students are classified into five groups as susceptible, exposed, drug users, drug addicted and the recovered for the dynamical equations, the definitions of the variables and parameters used in the model are given in the Table 1 below.

$$\frac{dS}{dt} = \Lambda - \beta S(t)D(t)\big(1+\alpha D(t)\big) - (\mu+\omega)S(t); \; S(0) = S_0 \tag{1}$$

$$\frac{dE}{dt} = \omega S(t) - (\mu+\gamma)E(t); \; E(0) = E_0 \tag{2}$$

$$\frac{dD}{dt} = \gamma E(t) + \beta S(t)D(t)\big(1+\alpha D(t)\big) - (\sigma+\delta+p+\mu)D(t) + \rho R(t); \; D(0) = D_0 \tag{3}$$

$$\frac{dA}{dt} = \sigma D(t) - (\theta+\tau+\mu)A(t); \; A(0) = A_0 \tag{4}$$

$$\frac{dR}{dt} = \tau A(t) + \delta D(t) - (\mu+\rho)R(t); \; R(0) = R_0 \tag{5}$$

Table 1: Definitions of parameters and variables used in the mathematical model

Parameters	Definitions	Hypothetical Values	Source
Λ.	recruitment rate into the susceptible population	0.8	[9]
β.	transmission rate of susceptible students into the illicit drug users	0.03	[9]
α.	rate of imitation into the use of illicit drugs among the students	0.257	Assumed
ω.	rate of susceptible being exposed to the use of the illicit drugs	0.47	Assumed
μ.	Natural death rate of the students at different groups	0.02	[1]
γ.	rate at which exposed students are got into the use of the illicit drugs	0.3	Assumed
σ.	rate at which illicit drug users are addicted to drugs	0.07	Assumed
δ.	Natural recovery rate of the illicit drug users through abstinence from the use of drugs	0.5	Assumed
p.	rate at which death occurs among the illicit drug users	0.179	Assumed
ϱ.	rate at which recovered students are returned to the use of illicit drugs after recovery	0.046	[9]
θ.	rate at which death occurs among the drug addicts	0.0182	Assumed
τ.	rate at which drug addicts are recovered from the use of illicit drugs	0.28	Assumed
S(t)	Rate of Susceptible students to illicit drug use at time t.	0.8000	Estimated
E(t)	Rate of Exposed students to illicit drug use at time t.	0.1120	Estimated
D(t)	Rate of Illicit Drug users among the students at time t.	0.0380	Estimated
A(t)	Rate of Drug Addicts among the students at time t.	0.0330	Estimated
R(t)	Rate of Recovered Students from the illicit drug users and drug addicts at time t.	0.0170	Estimated
N(t)	Total population of students considered in fractional form	1.0000	Estimated

1.2 *Concept of Eigenvalue Elasticity and Sensitivity Analyses*

Eigenvalue elasticities measure the transient – response sensitivities of the model to parameters and since the values of elasticities are dimensionless, they can be compared with each other. This can aid us identifying the parameters which could greatly influence the mathematical model [10].

(a) Eigenvalue Sensitivity with respect to a parameter:
This is defined as the partial derivative of the eigenvalue with respect to that parameter [10]. The eigenvalue sensitivity S_i (i = 1, - - -, N and N is the dimension of the state vector) with respect to the j^{th} parameter of the system p_j (j = 1, 2, - - -, 12) is given in the form;

$$S_i(p_j) = \lim_{\Delta p_j \to 0} \frac{\Delta\lambda_i}{\Delta p_j} = \frac{\partial\lambda_i}{\partial p_j} = I_i^T \frac{\partial J}{\partial p_j} r_i \quad (6)$$

(b) Eigenvalue Elasticity with respect to a parameter:
This is defined as the partial derivative of the eigenvalue with respect to that parameter normalized for the size of the parameter and the size of the eigenvalue [10]. This could also be described as the product of the eigenvalue sensitivity and the ratio of the eigenvalue and parameter [1]. Thus, it is given in the form;

$$E_i(p_j) = \lim_{\Delta p_j \to 0} \frac{\frac{\Delta\lambda_i}{\lambda_i}}{\frac{\Delta p_j}{p_j}} = \frac{\frac{\partial\lambda_i}{\lambda_i}}{\frac{\partial p_j}{p_j}} = \frac{\partial\lambda_i}{\partial p_j} \cdot \frac{p_j}{\lambda_i} = I_i^T \frac{\partial J}{\partial p_j} r_i . \frac{p_j}{\lambda_i} \quad (7)$$

With these equations (6) and (7), the eigenvalue elasticity and sensitivity with respect to a parameter can be computed using the left eigenvectors (I_i) and the right eigenvectors (r_i) with the partial derivatives of the linearized Jacobian matrix (J) with respect to a parameter (p_j). Because J and $\frac{\partial J}{\partial p_j}$ can often be easily determined symbolically and because the eigenvalues can be computed for particular parameter values and points in time, both eigenvalue elasticity and sensitivity with respect to a parameter can be computed without the need to either compute closed form expressions for eigenvalues or to perform numeric differentiation [8].
Using the MATLAB software package, the computer program was written for the evaluation of the values of eigenvalue elasticity and sensitivity of the mathematical model given in equations (1) – (5)

and the values were presented on the Table 2. The results obtained are shown in the table below and the computer program is shown under the appendix.

(c) Parameter with the Greatest Impact:
Using both the eigenvalue elasticity and sensitivity analysis formular with respect to a parameter, the Table 2 below shows the results obtained from the computer program written using the MATLAB software.

Table 2: Eigenvalue Sensitivity and Elasticity Values for each parameter

S/N	Parameters	Eigenvalue Sensitivity	Eigenvalue Elasticity
1	Beta (β)	0.02621710384	0.098686931
2	d	0.00047340115	0.00002257182
3	alpha (α)	0.00005852097	0.00000188711
4	mu (μ)	-1.0000000000	-0.02509479616
5	omega (ω)	-0.10971895	-0.06470430839
6	S	0.0009794547	0.00098316864
7	gamma (γ)	-0.11063846226	-0.04164674488
8	sigma (σ)	-0.82979834823	-0.07288267142
9	delta (δ)	-0.91571776271	-0.57449376499
10	P	-0.86150384922	-0.19349240825
11	rho (ϱ)	-0.57508111803	-0.0331925499
12	theta (θ)	0.00463552033	0.00010585797
13	tau (τ)	0.01256189558	0.00441333493

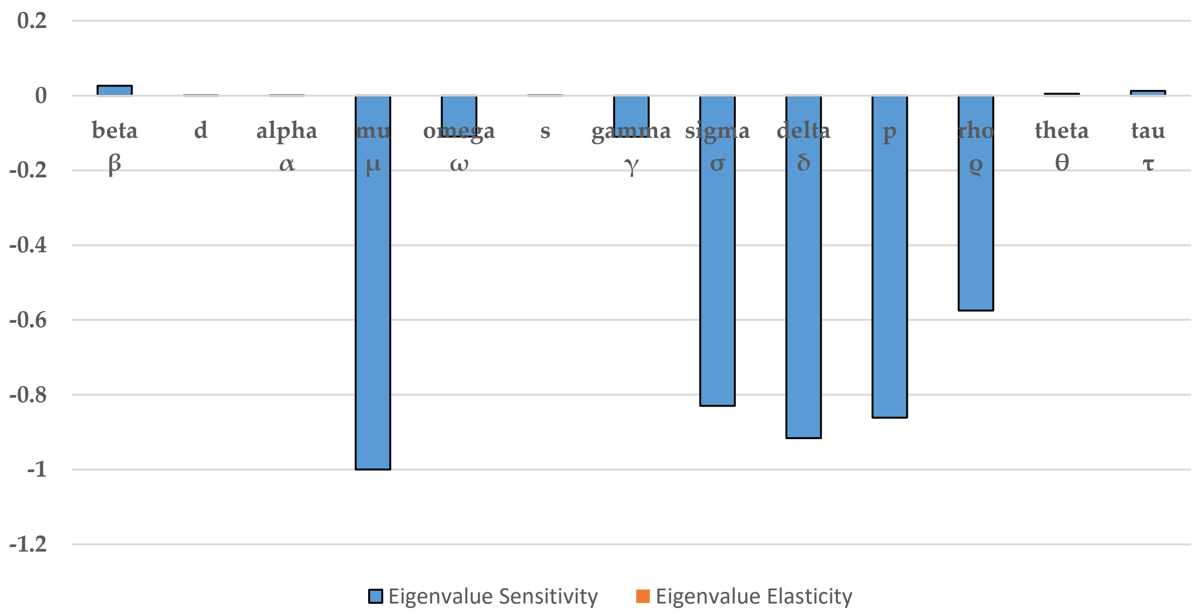

Figure 1: The Chart that shows the Eigenvalue Sensitivity and Elasticity Values.

2. Results and Discussion
From the above Table 2 and Figure 1, beta β **(transmission rate of susceptible students into the illicit drug users)** has the highest value. This parameter needs to be looked into by the parents of the students, government and the policy makers of the tertiary institutions in order to reduce or eradicate the drug addiction or drug abuse in our institutions. With this detected parameter, it is helpful for everyone to discourage our students from the use of illicit drugs in our tertiary institutions in order to reduce or eradicate the transmission rate of the use of illicit drugs amongst our students in tertiary institutions.

3. Conclusion

From the above study, we could see that mathematical modelling can solve real life problems especially in solving the problem of drug abuse and drug addiction among students of tertiary institutions in Nigeria. Drug addiction can be prevented and controlled, it is necessary, therefore, to help youths perceive drug abuse as harmful and they should reduce or stop their drug taking. In addition, it is necessary to help youths and the general public to understand the risks of drug abuse and also teachers, parents and health care professionals should keep sending the messages that, drug addiction can be prevented if a person never abuses drugs.

4. Recommendation

In order to reduce the transmission rate of susceptible students into the illicit drug users, the following recommendations are suggested for necessary actions. They are:

(i) Teaching and awareness programs for the youth on the abstains from the use of illicit drugs should be organized regularly to address the type of drug abuse problems in our tertiary institutions;
(ii) Our students should try and resist peer pressure from the use of illicit drugs;
(iii) Our students need to learn how to manage stress and anxiety instead of engaging in the use of illicit drugs;
(iv) Government should increase taxes on addictive materials like cigarettes such that our students will not be able to have access to the use of illicit drugs;
(v) Role of parents is imperative in the reduction of rate of transmission of the use of illicit drugs among our students. Parents should always check on their wards while in school;
(vi) Easy and affordable access to rehabilitation centres in the country for the addictive drug users.

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Supplementary Data

MATLAB Codes for Computation

```
matdrug = [-0.49115 0 -0.024469 0 0;0.47 -0.32 0 0 0;0.0011511 0.3 -0.74453 0 0.046;0 0 0.07 -0.3182 0;0 0 0.5 0.28 -0.066]

[wmatdrug, dmatdrug] = eig(matdrug)
vmatdrug = conj(inv(wmatdrug))
lambdiag = diag(dmatdrug)
lambvec = sort(abs(lambdiag))
dim = size(lambvec,1)
lambda1 = lambvec(dim)
lambda2 = lambvec(dim - 1)
rho = lambda1/lambda2
t20 = log(20)/log(rho)
imax = find(lambdiag==max(lambdiag));

syms beta d alpha mu omega s gamma sigma delta p rho theta tau;

matdrug =[-beta*d-alpha*beta*d*d-(mu+omega) 0 -beta*s-2*alpha*beta*s*d 0 0;omega -(mu+gamma) 0 0 0;beta*d+alpha*beta*d*d gamma beta*s+2*alpha*beta*s*d-(sigma+delta+p+mu) 0 rho;0 0 sigma -(theta+tau+mu) 0;0 0 delta tau -(mu+rho)]

PD_beta = diff(matdrug, beta)
PD_d = diff(matdrug, d)
PD_alpha = diff(matdrug, alpha)
PD_mu = diff(matdrug, mu)
PD_omega = diff(matdrug, omega)
PD_s = diff(matdrug, s)
PD_gamma = diff(matdrug, gamma)
PD_sigma = diff(matdrug, sigma)
PD_delta = diff(matdrug, delta)
PD_p = diff(matdrug, p)
PD_rho = diff(matdrug, rho)
PD_theta = diff(matdrug, theta)
PD_tau = diff(matdrug, tau)

beta =0.03; d = 0.038; alpha = 0.257; mu = 0.02; omega = 0.47; s = 0.8; gamma = 0.3; sigma = 0.07; delta = 0.5; p = 0.179; rho = 0.046; theta = 0.0182; tau = 0.28;

matdrug = subs(matdrug)
PD_beta = subs(PD_beta)
PD_d = subs(PD_d)
PD_alpha = subs(PD_alpha)
PD_mu = subs(PD_mu)
PD_omega = subs(PD_omega)
PD_s = subs(PD_s)
PD_gamma = subs(PD_gamma)
PD_sigma = subs(PD_sigma)
PD_delta = subs(PD_delta)
PD_p = subs(PD_p)
PD_rho = subs(PD_rho)
PD_theta = subs(PD_theta)
PD_tau = subs(PD_tau)

S_beta = sum(sum(PD_beta.*sens))
S_d = sum(sum(PD_d.*sens))
S_alpha = sum(sum(PD_alpha.*sens))
S_mu = sum(sum(PD_mu.*sens))
S_omega = sum(sum(PD_omega.*sens))
S_s = sum(sum(PD_s.*sens))
```

```
S_gamma = sum(sum(PD_gamma.*sens))
S_sigma = sum(sum(PD_sigma.*sens))
S_delta = sum(sum(PD_delta.*sens))
S_p = sum(sum(PD_p.*sens))
S_rho = sum(sum(PD_rho.*sens))
S_theta = sum(sum(PD_theta.*sens))
S_tau = sum(sum(PD_tau.*sens))

E_beta = beta/lambda1*S_beta,          E_d = d/lambda1*S_d,     E_alpha = alpha/lambda1*S_alpha,
E_mu = mu/lambda1*S_mu,                E_omega = omega/lambda1*S_omega
E_s = s/lambda1*S_s,                   E_gamma = gamma/lambda1*S_gamma,
E_sigma = sigma/lambda1*S_sigma,       E_delta = delta/lambda1*S_delta
E_p = p/lambda1*S_p,                   E_rho = rho/lambda1*S_rho,
E_theta = theta/lambda1*S_theta,
E_tau = tau/lambda1*S_tau
```

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